Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY
Summer Examination-2019

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT2 **Branch: Diploma (All)**

Semester : 2 **Date : 20/04/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:
(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) $\lim_{x\to 0}\frac{\sin 3x}{2x}$ = _____

(A) 3/2 (B) 2/3 (C) 1/3 (D) 1/2

b) $\lim_{x\to a}\frac{x^n-a^n}{x-a}$ = _____

(A) ax^{n-1} (B) nx^{n-1} (C) na^{n-1} (D) None of these

c) $\lim_{x\to 0}\left(1+\frac{2}{x}\right)^x$ = _____

(A) e^2 (B) e (C) $e^{1/2}$ (D) None of these

d) If $f(x)=\sin x$ then $f(2\pi)$ =_____

(A) 1 (B) 0 (C) –1 (D) 2

e) $\frac{d\left(e^{7x}\right)}{dx}$ = _____

(A) $7e^x$ (B) e^x (C) $\frac{e^{7x}}{7}$ (D) $7e^{7x}$

f) $\frac{d\left(\tan^{-1}x\right)}{dx}$ = _____

(A) $\frac{-1}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{1+x^2}$ (D) $\frac{-1}{1+x^2}$

g) $\frac{d\left(\cos x\right)}{dx}$ = _____

(A) $-\sec x$ (B) $\sec x$ (C) $-\sin x$ (D) $\sin x$

h) $\frac{d(\sin^2 x+\cos^2 x)}{dx}$ = _____

(A) 2 (B) 1 (C) 0 (D) None of these

i) $\int \frac{1}{\sqrt{x^2+4}}\,dx =$ _____

(A) $\cot^{-1}\frac{x}{2}+c$ (B) $\tan^{-1}\frac{x}{2}+c$ (C) $\log\left|x+\sqrt{x^2+4}\right|+c$

(D) none of these

j) $\int e^x\,dx =$ _____

(A) $\log x + c$ (B) $e^x + c$ (C) 1 (D) 0

k) $\int_0^1 \frac{2}{1+x^2}\,dx =$ _____

(A) π (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) None of these

l) Magnitude of $3i+4j+5k$ is _____.

(A) $2\sqrt{5}$ (B) 0 (C) -6 (D) $5\sqrt{2}$

m) If $\bar{x}=(1,\ 1,\ 1)$ and $\bar{y}=(2,\ -1,\ 3)$ then $\bar{x}\times\bar{y}=$ _____

(A) $(4,-1,3)$ (B) $(-4,-1,3)$ (C) $(-4,1,3)$ (D) $(4,-1,-3)$

n) If $(1,-2,3)\cdot(4,5,k)=0$ then $k=$ _____.

(A) -2 (B) 2 (C) 1/2 (D) None of these

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) If $\bar{a} = 3i - j - 4k$, $\bar{b} = -2i + 4j - 3k$ and $\bar{c} = -i + 2j - 5k$ then find $\left|\bar{a} + 2\bar{b} - \bar{c}\right|$. **(5)**

b) Find unit vector which is perpendicular to $\bar{a} = i + j + k$ and $\bar{b} = 2i - 2j + k$. **(5)**

c) Evaluate: $\lim_{n\to\infty}\frac{1^3+2^3+.....+n^3}{n^2(1+2+.....+n)}$ **(4)**

Q-3 **Attempt all questions** **(14)**

a) Prove that $\lim_{x\to\infty}\left[\sqrt{x}\left(\sqrt{x+p}-\sqrt{x}\right)\right]=\frac{p}{2}$. **(5)**

b) Evaluate: $\lim_{\theta\to 0}\frac{\cos ec\theta-\cot\theta}{\theta}$ **(5)**

c) Find $\frac{dy}{dx}$ if $y=\frac{\log x}{x}$ at $x = 1$. **(4)**

Q-4 **Attempt all questions** **(14)**

a) The equation of motion of a particle is $S=2t^3+3t^2-12t+5$. **(5)**
(i) Find velocity at $t = 0$. (ii) Find acceleration at $t = 1$.

b) Find $\frac{dy}{dx}$ if $y=\log\sqrt{\frac{1+\sin x}{1-\sin x}}$. **(5)**

c) Simplify: $(10i + 2j + 3k)\cdot[(i - 2j + 2k)\times(3i - 2j - 2k)]$. **(4)**

Q-5 **Attempt all questions** **(14)**

a) Prove that angle between two vectors $i + 2j$ and $i + j + 3k$ is $\sin^{-1}\left(\sqrt{\frac{46}{55}}\right)$. **(5)**

b) If $y = e^x \sin x$ then prove that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$. **(5)**

c) Evaluate: $\lim_{x\to 2}\frac{x^7 - 128}{x^4 - 16}$ **(4)**

Q-6 **Attempt all questions** **(14)**

a) Constant forces $3i - j + 2k$ and $i + 3j - k$ act on a particle and the particle moves the point $2i + 3j + k$ to the point $5i + 2j + 3k$. Find the work done by the forces. **(5)**

b) Prove that $\int_0^{\frac{\pi}{2}} \frac{\sec x}{\sec x + \operatorname{cosec} x}\,dx = \frac{\pi}{4}$. **(5)**

c) Find derivative of $y = 3^{4x}$ using first principle. **(4)**

Q-7 **Attempt all questions** **(14)**

a) Evaluate: $\int\left[\sqrt{1+\sin 2x} + \sqrt{\frac{1+\cos 2x}{1-\cos 2x}}\right]dx$ **(5)**

b) Find the area of the region bounded between curve $y = x^2$ and straight – line $x = 2$. **(5)**

c) For what value of *p*, vectors $2i + 3j - k$ and $pi - j + 3k$ are perpendicular to each other? **(4)**

Q-8 **Attempt all questions** **(14)**

a) Evaluate: $\int x^2 \log x\,dx$ **(5)**

b) Find $\frac{dy}{dx}$ if $y = x^{\sin^2 x}$. **(5)**

c) Evaluate: $\int_0^{10} W\,dx$ *Where* $W = \frac{3}{4}x\left(1 + \frac{x}{10}\right)$ **(4)**